

Two-photon decay of pseudoscalar quarkonia

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Abstract.

We report on our recent evaluation of the two-photon width of the pseudoscalar quarkonia, $\eta_c(nS)$ and $\eta_b(nS)$ in an approach based on Heavy-Quark Spin Symmetry (HQSS). To what concerns the $1S$ state η_c , our parameter-free computation agrees with experiments, as well as most of other theoretical works. On the other hand, our computation for the $2S$ -state looks $2S$ like a confirmation that there may exist an anomaly related to the decay of η'_c , especially in the light of the new preliminary result of the Belle collaboration. We also point out that the essentially model-independent ratio of η_b two-photon width to the Υ leptonic width and the η_b two-photon width could be used to extract the strong coupling constant α_s .

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1. INTRODUCTION

Since the J/ψ discovery, bound states of heavy quarks –the heavy quarkonia– are expected to provide physicists with ideal means to study the main properties of QCD. With the time though, it appeared progressively that such quark systems are not so easy to understand and controversies about their production mechanisms are still going on [1, 2]. Fortunately, the physics involved in their decay seems to be rather well understood within the conventional framework of QCD [1, 3].

However, recently two experimental estimations of the two-photon width of the η'_c , one published by the CLEO collaboration [4] ($\Gamma_{\gamma\gamma}(\eta'_c) = 1.3 \pm 0.6$ keV) and another preliminary by the Belle collaboration ($\Gamma_{\gamma\gamma}(\eta'_c) = 0.59 \pm 0.13 \pm 0.14$ keV) [5, 6] contradict most of the existing theoretical predictions lying in the range 3.7 to 5.7 keV [7, 8, 9, 10, 11, 12, 13, 14]. This is rather surprising since such electromagnetic decays of non-relativistic states should be rather easy to describe from a theoretical point of view.

It was therefore our purpose in [15, 16] to reanalyse such a process from a very basic starting point: the heavy-quark spin symmetry (HQSS). Indeed, in a non-

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relativistic system, the difference between the $1S$ and $2S$ widths would appear only at the level of the wave function at the origin. In virtue of such a symmetry, both wave functions should not differ much from the ones of 3S_1 state, which are a priori well known from the leptonic decays of J/ψ and ψ' .

We report here on our effective approach based on HQSS and compare its results with other available calculations. Since none of them is able to predict both experimental measurements for $\Gamma_{\gamma\gamma}(\eta_c)$ and $\Gamma_{\gamma\gamma}(\eta'_c)$, we also discuss some of the hypotheses made by CLEO and Belle for the extraction of those widths from their experimental observables. We finally report on the corresponding predictions for $\eta_b(nS)$ states.

2. EFFECTIVE LAGRANGIAN FOR 1S_0 DECAY INTO TWO PHOTONS

In the two-photon decay of a heavy quarkonium bound state, the outgoing-photon momentum is large compared to the relative momentum of the quark-antiquark bound state, which is $\mathcal{O}(\Lambda/m_Q)$, with $\Lambda \ll m_Q$. One obtains an effective Lagrangian for the process $Q\bar{Q} \rightarrow \gamma\gamma$ (represented by the first diagram in Fig.1) by expanding the heavy-quark propagator in powers of q^2/m_Q^2 , and neglecting $\mathcal{O}(q^2/m_Q^2)$ terms ($q = p_Q - p_{\bar{Q}}$). Like leptonic decay, the two-photon decay, in this approximation, is described by the following effective Lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\gamma\gamma} &= -ic_1(\bar{Q}\gamma^\sigma\gamma_5 Q)\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}A^\rho \\ \mathcal{L}_{\text{eff}}^{\ell\bar{\ell}} &= -c_2(\bar{Q}\gamma^\mu Q)(\ell\gamma_\mu\bar{\ell})\end{aligned}\tag{1}$$

with

$$c_1 \simeq \frac{Q_Q^2(4\pi\alpha_{\text{em}})}{M_{1S_0}^2 + b_{1S_0}M_{1S_0}}, \quad c_2 = \frac{Q_Q(4\pi\alpha_{\text{em}})}{M_{3S_1}^2}.\tag{2}$$

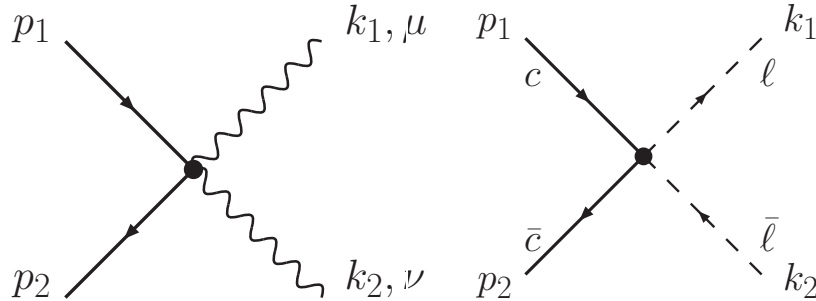


FIGURE 1. Effective coupling between a $Q\bar{Q}$ and two photons (left) and a lepton pair (right)

The factor $1/(M_{1S_0}^2 + b_{1S_0}M_{1S_0})$ in c_1 contains the binding-energy effects (the binding-energy b is defined as $b = 2m_Q - M$) and is obtained from the denominator

of the heavy-quark propagator (k_1, k_2 being the outgoing-photon momenta):

$$\frac{1}{[(k_1 - k_2)^2/4 - m_Q^2]} . \quad (3)$$

The decay amplitude is then given by the matrix element of the axial-vector current $\bar{Q}\gamma^\mu\gamma_5 Q$ similar to the quarkonium leptonic decay amplitude which is given by the vector-current matrix element $\bar{Q}\gamma^\mu Q$ for $^3S_1 \rightarrow \ell^+\ell^-$. Thus for decays of S -wave quarkonium into two photons or a dilepton pair $\ell\bar{\ell}$, we have:

$$\begin{aligned} \mathcal{M}_{\ell\bar{\ell}} &= Q_Q(4\pi\alpha_{\text{em}})\frac{f_{^3S_1}}{M_{^3S_1}}\varepsilon_\mu(\ell\gamma^\mu\bar{\ell}) \\ \mathcal{M}_{\gamma\gamma} &= -4iQ_Q^2(4\pi\alpha_{\text{em}})\frac{f_{^1S_0}}{M_{^1S_0}^2 + b_{^1S_0}M_{^1S_0}}\varepsilon_{\mu\nu\rho\sigma}\varepsilon_1^\mu\varepsilon_2^\nu k_1^\rho k_2^\sigma \end{aligned} \quad (4)$$

where

$$\langle 0|\bar{Q}\gamma^\mu Q|^3S_1\rangle = f_{^3S_1}M_{^3S_1}\varepsilon^\mu, \quad \langle 0|\bar{Q}\gamma^\mu\gamma_5 Q|^1S_0\rangle = if_{^1S_0}P^\mu. \quad (5)$$

from which the decay rates are:

$$\Gamma_{\ell\bar{\ell}}(^3S_1) = \frac{4\pi Q_Q^2\alpha_{\text{em}}^2 f_{^3S_1}^2}{3M_{^3S_1}}, \quad \Gamma_{\gamma\gamma}(^1S_0) = \frac{4\pi Q_Q^4\alpha_{\text{em}}^2 f_{^1S_0}^2}{M_{^1S_0}}. \quad (6)$$

By taking $M_{^3S_1}f_{^3S_1}^2 = 12|\psi(0)|^2$, we recover the usual non-relativistic expression for the decay rate which, whose NLO QCD radiative corrections are given by

$$\Gamma^{NLO}(^3S_1) = \Gamma^{LO}(^3S_1)\left(1 - \frac{\alpha_s}{\pi}\frac{16}{3}\right) \quad (7)$$

$$\Gamma^{NLO}(^1S_0) = \Gamma^{LO}(^1S_0)\left(1 - \frac{\alpha_s}{\pi}\frac{(20 - \pi^2)}{3}\right). \quad (8)$$

3. MATRIX ELEMENTS OF LOCAL OPERATORS

We have shown that in the approximation of neglecting $\mathcal{O}(q^2/m_Q^2)$ terms, the two-photon decay amplitude is given by the 1S_0 decay constant $f_{^1S_0}$. We now derive a symmetry relation between $f_{^1S_0}$ and $f_{^3S_1}$, the 3S_1 leptonic decay constant using the relativistic spin projection operators for a relativistic Bethe-Salpeter quarkonium bound state [17].

Consider now the matrix elements of local operators in a fermion-antifermion system with a given spin S and orbital angular momentum L [18, 19] :

$$\mathcal{A} = \int \frac{d^4q}{(2\pi)^4} \text{Tr} \mathcal{O}(0)\chi(P, q) \quad (9)$$

P is the total 4-momentum of the quarkonium system, q is the relative 4-momentum between the quark and anti-quark and $\chi(P, q)$ is the Bethe-Salpeter wave function.

For a quarkonium system in a fixed total, orbital and spin angular momentum, $\chi(P, q)$ is given by (\mathbf{q} is the relative 3-momentum vector of q)

$$\begin{aligned}\chi(P, q; J, J_z, L, S) &= \sum_{M, S_z} 2\pi \delta(q^0 - \frac{\mathbf{q}^2}{2m}) \psi_{LM}(\mathbf{q}) \langle LM; SS_z | JJ_z \rangle \\ &\times \sqrt{\frac{3}{m}} \sum_{s, \bar{s}} u(P/2 + q, s) \bar{v}(P/2 - q, \bar{s}) \langle \frac{1}{2}s; \frac{1}{2}\bar{s} | SS_z \rangle \\ &= \sum_{M, S_z} 2\pi \delta(q^0 - \frac{\mathbf{q}^2}{2m}) \psi_{LM}(\mathbf{q}) \mathcal{P}_{SS_z}(P, q) \langle LM; SS_z | JJ_z \rangle . \quad (10)\end{aligned}$$

The spin projection operators $\mathcal{P}_{SS_z}(P, q)$ are

$$\begin{aligned}\mathcal{P}_{0,0}(P, q) &= \sqrt{\frac{3}{8m^3}} [-(P/2 + \not{q}) + m] \gamma_5 [(P/2 - \not{q}) + m] \\ \mathcal{P}_{1,S_z}(P, q) &= \sqrt{\frac{3}{8m^3}} [-(P/2 + \not{q}) + m] \not{\epsilon}(S_z) [(P/2 - \not{q}) + m] . \quad (11)\end{aligned}$$

For S -wave quarkonium in a spin singlet $S = 0$ and spin triplet $S = 1$ state:

$$\mathcal{A}^{(2S+1)S_J} = \text{Tr}(\mathcal{O}(0) \mathcal{P}_{JJ_z}(P, 0)) \int \frac{d^3 q}{(2\pi)^3} \psi_{00}(q) . \quad (12)$$

In this expression the q -dependence in the spin projection operator has been dropped and the integral in Eq.(12) is the S -wave function at the origin [19]:

$$\int \frac{d^3 q}{(2\pi)^3} \psi_{00}(q) = \frac{1}{\sqrt{4\pi}} \mathcal{R}_0(0) . \quad (13)$$

Using Eq.(11) and Eq.(12) to compute the matrix elements $\langle 0 | \bar{Q} \gamma_\mu \gamma_5 Q | P \rangle$ and $\langle 0 | \bar{Q} \gamma_\mu Q | V \rangle$ for the singlet $S = 0$ pseudo-scalar meson P and for the triplet $S = 1$ vector meson V , we find, neglecting quadratic $\mathcal{O}(q^2)$ terms.

$$f_P = \sqrt{\frac{3}{32\pi m^3}} \mathcal{R}_0(0) (4m) , \quad f_V = \sqrt{\frac{3}{32\pi m^3}} \mathcal{R}_0(0) \frac{(M^2 + 4m^2)}{M} \quad (14)$$

Thus we get the relation

$$f_{1S_0} \simeq f_{3S_1} + \mathcal{O}(b^2/M^2). \quad (15)$$

It is expected that this relation holds also for excited state of charmonium and bottomonium where the binding terms $\mathcal{O}(b^2/M^2)$ can be neglected. This is a manifestation of heavy-quark spin symmetry(HQSS). In this limit, the two-photon width of singlet $S = 0$ quarkonium state can be obtained from the leptonic width of triplet $S = 0$ quarkonium state without using a bound state description. This approach differs from the traditional non-relativistic bound state approach in the

use of local operators for which the matrix elements could be measured or extracted from physical quantities, or computed from QCD sum rules [20, 21] and lattice QCD [22].

The ratio of the η_c two-photon width to J/ψ leptonic width in the limit of HQSS is then:

$$R_{\eta_c} = \frac{\Gamma_{\gamma\gamma}(\eta_c)}{\Gamma_{\ell\bar{\ell}}(J/\psi)} = 3Q_c^2 \frac{M_{J/\psi}}{M_{\eta_c}} \left(1 + \frac{\alpha_s}{\pi} \frac{(\pi^2 - 4)}{3} \right). \quad (16)$$

For $M_{\eta_c} = M_{J/\psi}$, the above expression becomes the usual non-relativistic result [23, 24] as mentioned above. From the measured J/ψ leptonic width, we get $\Gamma_{\gamma\gamma}(\eta_c) = 7.46$ keV. Including NLO QCD radiative corrections with $\alpha_s = 0.26$, we find $\Gamma_{\gamma\gamma}(\eta_c) = 9.66$ keV in near agreement with the world average value $7.4 \pm 0.9 \pm 2.1$ keV. A similar result is obtained in [24] which gives $8.16 \pm 0.57 \pm 0.04$ keV.

Thus the effective Lagrangian approach successfully predicts the η_c two-photon width in a simple, essentially model-independent manner.

4. HQSS PREDICTIONS FOR $\Gamma_{\gamma\gamma}(\eta'_c)$

To obtain the prediction for η'_c , we shall apply HQSS to $2S$ states. Thus, assuming $f_{\eta'_c} = f_{\psi'}$, and neglecting binding-energy terms, we find: $\Gamma_{\gamma\gamma}(\eta'_c) = \Gamma_{\gamma\gamma}(\eta_c) \frac{f_{\psi'}^2}{f_{J/\psi}^2} = 3.45$ keV. This value is more than twice the evaluation by CLEO and five times the one by Belle. Our results is however nearly in agreement with other theoretical calculations [7, 8, 9] as shown in Table 1. Other approaches [10, 11, 12, 14] seem to be closer to the latter measurements but then undershoot clearly the measurements for η_c .

Including binding-energy terms, for $M_{\eta_c} \simeq M_{J/\psi}$, $M_{\eta'_c} \simeq M_{\psi'}$, we have

$$\Gamma_{\gamma\gamma}(\eta'_c) = \Gamma_{\gamma\gamma}(\eta_c) \left(\frac{1 + b_{\eta_c}/M_{\eta_c}}{1 + b_{\eta'_c}/M_{\eta'_c}} \right)^2 \times \left(\frac{\Gamma_{e^+e^-}(\psi')}{\Gamma_{e^+e^-}(J/\psi)} \right) \quad (17)$$

which gives

$$\Gamma_{\gamma\gamma}(\eta'_c) = 4.1 \text{ keV} . \quad (18)$$

TABLE 1. Theoretical predictions for $\Gamma_{\gamma\gamma}(\eta_c)$ and $\Gamma_{\gamma\gamma}(\eta'_c)$. (All values are in units of keV).

$\Gamma_{\gamma\gamma}$	This work	[7]	[8]	[9]	[10]	[11]	[12]	[14]
η_c	7.5 – 10	4.8	7.14 ± 0.95	$11.8 \pm 0.8 \pm 0.6$	3.5 ± 0.4	5.5	5.5	6.2
η'_c	3.5 – 4.5	3.7	4.44 ± 0.48	$5.7 \pm 0.5 \pm 0.6$	1.38 ± 0.3	2.1	1.8	3.36-1.95

Binding-energy corrections seem then to worsen comparison with data and maybe point at anomaly in the decay $\eta_c(nS) \rightarrow \gamma\gamma$. However, before drawing such a conclusion it is necessary to discuss the experimental hypotheses made for the extraction of the aforementioned widths.

5. $\mathcal{B}(\eta_c(nS) \rightarrow KK\pi)$

The measured value from CLEO [4] :

$$\Gamma_{\gamma\gamma}(\eta'_c) = 1.3 \pm 0.6 \text{ keV}, \quad (19)$$

was effectively done by considering the following quantity :

$$R(\eta'_c/\eta_c) = \frac{\Gamma_{\gamma\gamma}(\eta'_c) \times \mathcal{B}(\eta'_c \rightarrow K_S K \pi)}{\Gamma_{\gamma\gamma}(\eta_c) \times \mathcal{B}(\eta_c \rightarrow K_S K \pi)} = 0.18 \pm 0.05 \pm 0.02. \quad (20)$$

To obtain $\Gamma_{\gamma\gamma}(\eta'_c)$ from the above data, they made the assumption that

$$\mathcal{B}(\eta'_c \rightarrow K_S K \pi) \approx \mathcal{B}(\eta_c \rightarrow K_S K \pi) \quad (21)$$

and in turn found the result of Eq. (19).

Such an assumption is in fact supported by a couple of observations. Belle measurements of $B \rightarrow \eta_c K$ and $B \rightarrow \eta'_c K$ gives [26]:

$$R(\eta'_c K/\eta_c K) = \frac{\mathcal{B}(B^0 \rightarrow \eta'_c K^0) \times \mathcal{B}(\eta'_c \rightarrow K_S K^+ \pi^-)}{\mathcal{B}(B^0 \rightarrow \eta_c K^0) \times \mathcal{B}(\eta_c \rightarrow K_S K^+ \pi^-)} = 0.38 \pm 0.12 \pm 0.05. \quad (22)$$

Using the approximate equality Eq. (21), one would obtain

$$\frac{\mathcal{B}(B^0 \rightarrow \eta'_c K^0)}{\mathcal{B}(B^0 \rightarrow \eta_c K^0)} \approx 0.4, \quad (23)$$

which agrees more or less with the QCD factorization (QCDF) prediction [27] :

$$\frac{\mathcal{B}(B^0 \rightarrow \eta'_c K^0)}{\mathcal{B}(B^0 \rightarrow \eta_c K^0)} \approx 0.9 \times \left(\frac{f_{\eta'_c}}{f_{\eta_c}}\right)^2 \approx 0.45. \quad (24)$$

On the other hand, it is expected from $SU(2)$ flavor symmetry that one would have the approximate equality between the ratios

$$\frac{\mathcal{B}(B^0 \rightarrow \eta'_c K^0)}{\mathcal{B}(B^0 \rightarrow \eta_c K^0)} \approx \frac{\mathcal{B}(B^+ \rightarrow \eta'_c K^+)}{\mathcal{B}(B^+ \rightarrow \eta_c K^+)}. \quad (25)$$

This is indeed the case since the following ratio obtained from BABAR [28]

$$\frac{\mathcal{B}(B^+ \rightarrow \eta'_c K^+)}{\mathcal{B}(B^+ \rightarrow \eta_c K^+)} = 0.38 \pm 0.25 \quad (26)$$

corresponds again to Eq. (23).

Those observations therefore tend to support the assumption of the approximate equality between the $\eta'_c \rightarrow KK\pi$ and $\eta_c \rightarrow KK\pi$ branching ratio. This would confirm a small $\eta'_c \rightarrow \gamma\gamma$ decay rate as quoted above.

Albeit, we also note that the good agreement with QCDF predictions for the measured ratio $\mathcal{B}(B^0 \rightarrow \eta'_c K^0)/\mathcal{B}(B^0 \rightarrow \eta_c K^0)$ and $\mathcal{B}(B^+ \rightarrow \eta'_c K^+)/\mathcal{B}(B^+ \rightarrow \eta_c K^+)$ at Belle and BABAR suggests that $f_{\eta'_c}/f_{\eta_c} \approx f_{\psi'}/f_{J/\psi}$, which in turn supports HQSS and our predicted value for the η'_c two-photon width which is more than twice bigger than the CLEO estimated value shown above. More precisely, comparing $R(\eta'_c/\eta_c)$ with $R(\eta'_c K/\eta_c K)$ and using QCDF value given in Eq.(24), we find

$$R(\eta'_c/\eta_c) \approx R(\eta'_c K/\eta_c K)/0.9 . \quad (27)$$

The Belle data in Eq. (22) would then implies $R(\eta'_c/\eta_c) \approx 0.42 \pm 0.13 \pm 0.05$, twice bigger than the CLEO data shown in Eq. (20).

Since QCD gives $\mathcal{B}(\eta_c \rightarrow \gamma\gamma) \approx \mathcal{B}(\eta'_c \rightarrow \gamma\gamma)$ and the predicted $\mathcal{B}(\eta_c \rightarrow \gamma\gamma)$ agrees well with experiments, we expected that the large value for the measured η'_c total width would imply a large value for the η'_c two-photon width. Thus it is difficult to understand the very small recent Belle measured η'_c two-photon width.

6. HQSS PREDICTIONS FOR $\Gamma_{\gamma\gamma}(\eta_b)$ AND $\Gamma_{\gamma\gamma}(\eta'_b)$

Since the b -quark mass is significantly higher than the c -quark mass, the effective Lagrangian and HQSS approach should work better for bottomonia decays to leptons and photons. We thus have:

$$R_{\eta_b} = \frac{\Gamma_{\gamma\gamma}(\eta_b)}{\Gamma_{\ell\bar{\ell}}(\Upsilon)} = 3Q_b^2 \frac{M_\Upsilon}{M_{\eta_b}} \left(1 + \frac{\alpha_s}{\pi} \frac{(\pi^2 - 4)}{3} \right) \quad (28)$$

(neglecting the small b_{η_b}/M_{η_b} binding-energy term). This gives $\Gamma_{\gamma\gamma}(\eta_b) = 560 \text{ eV}$ ($\alpha_s(M_\Upsilon) = 0.16$, $M_{\eta_b} = 9300 \text{ MeV}$).

For η'_b and higher excited state, one has ($M_{\eta_b} \simeq M_\Upsilon$ and $M_{\eta'_b} \simeq M_{\Upsilon'}$):

$$\Gamma_{\gamma\gamma}(\eta'_b) = \Gamma_{\gamma\gamma}(\eta_b) \left(\frac{1 + b_{\eta_b}/M_{\eta_b}}{1 + b_{\eta'_b}/M_{\eta'_b}} \right)^2 \left(\frac{\Gamma_{e^+e^-}(\Upsilon')}{\Gamma_{e^+e^-}(\Upsilon)} \right). \quad (29)$$

which gives $\Gamma_{\gamma\gamma}(\eta'_b) = 250 \text{ eV}$ and $\Gamma_{\gamma\gamma}(\eta''_b) = 187 \text{ eV}$. In Table. 2 we give our prediction for the two-photon width of η_b , η'_b and η''_b together with other theoretical predictions. We note that our predicted values are somewhat higher than other predicted values.

Eq.(28) can be used to determine in a reliable way the value of α_s . The momentum scale at which α_s is to be evaluated here could be in principle be fixed with R_{η_b} .

Further check of consistency of the value for α_s may be possible in future measurements on the η_b and its two-photon decay branching ratio:

$$\frac{\Gamma_{\gamma\gamma}(\eta_b)}{\Gamma_{gg}(\eta_b)} = \frac{9}{2} Q_b^4 \frac{\alpha_{em}^2}{\alpha_s^2} \left(1 - 7.8 \frac{\alpha_s}{\pi} \right). \quad (30)$$

TABLE 2. Summary of theoretical predictions for $\Gamma_{\gamma\gamma}(\eta_b)$, $\Gamma_{\gamma\gamma}(\eta'_b)$ and $\Gamma_{\gamma\gamma}(\eta''_b)$. (All values are in units of eV).

$\Gamma_{\gamma\gamma}$	This work	[29]	[30]	[7]	[8]	[9]	[10]	[12]	[31]	[32]
η_b	560	460	230	170	384 ± 47	520	220 ± 40	350	214	466 ± 101
η'_b	269	200	70	-	191 ± 25	-	110 ± 20	150	121	-
η''_b	208	-	40	-	-	-	84 ± 12	100	90.6	-

7. CONCLUSION

We have shown here that effective Lagrangian approach and HQSS can be used to compute quarkonium decays into leptons and photons with relativistic kinematics, for both ground states and excited states of heavy-quarkonium systems.

We emphasised with our basically model-independent calculations that either HQSS holds for radially excited charmonia and there is a not-yet-understood specificity in the decay $\eta'_c \rightarrow \gamma\gamma$; either HQSS is strongly broken and this hints at large relativistic corrections at work in such decays. This would in turn explain why most of available models on the market are unable to give correct predictions for both η_c and η'_c decays.

Measurements of the two-photon widths for η_b and higher excited states could provide with a test for HQSS and a determination of the strong coupling constant α_s at a scale around the Υ mass, similarly to what has been done with the Υ leptonic width in the past.

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